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LETTER TO THE EDITOR

Geometrical dimensions characterizing quantum scattering through mesoscopic cavities

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Abstract. We report a heuristic result of statistical analysis of quantum scattering processes through mesoscopic conducting cavities. We elucidate the existence of geometrical dimensions involved in complicated conductance fluctuations. Our numerical results show that the number of dimensions, which is extractable experimentally from conductance fluctuations in the lowest transmittable mode, is related with non-integrability in the underlying dynamical systems.

The study of quantum transport through mesoscopic systems has become one of the 'hot' topics in recent years. An interesting problem for researchers in this field is a *chaotic scattering* event. From the viewpoint of 'quantum chaos' [1], it is highly desirable to examine the effect of integrability or non-integrability in classical dynamics on quantum transport in open systems.

A heuristic example of a mesoscopic open system is a two-dimensional conducting cavity whose characteristic size is a little larger than the Fermi wavelength of a scattering electron. The electron motion is ballistic, except for collisions with hard walls put along boundaries of the system, and the shape of the cavity directly determines the nature of classical dynamics of the inside electron. Because of the simplicity of the system, its quantum mechanics attracts attention both theoretically [2–5] and experimentally [6–9].

Scattering of an electron through a mesoscopic conducting cavity is equivalent to that of a microwave through a waveguide with a resonant cavity. Regardless of the integrability of the system, the conductance or the transmission coefficient shows noisy oscillations as a function of an external parameter. Recently, Baranger *et al* proposed a model using a random S-matrix theory to describe the universal features of the conductance of chaotic cavities [10, 11]. The model, including and excluding a prompt component, is based on the statistical ansatz for the S-matrix, assuming that the circular ensembles describe the S-matrix of a chaotic cavity. Their results for the ensemble average, variance, probability density of the conductance, and total channel-number dependence agree with a statistical analysis of the numerically obtained conductance of a chaotic cavity sampled along the energy axis in the presence and absence of a magnetic field. In contrast to closed systems [12], however, there is almost no analytical treatment for quantum transport through a *regular* cavity.

In this letter, we adopt another approach to the problem of quantum scattering to derive a quantity characterizing a wide variety of transport properties.

We start with a general situation where there is a mesoscopic conducting cavity with a pair of conducting leads. The system may be two or three dimensional and we do not necessarily restrict the shape of the cavity to be chaotic. Both of the leads are considered

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to have the same cross section. In classical dynamics, an electron incoming from a lead dwells for a while inside the cavity, bouncing within its boundaries, and then goes into a lead. If the electron has experienced many bounces, statistically it must be carrying some dynamical information about the cavity. To ensure this condition, we need to choose narrow leads with their orientation such that the contribution of direct trajectories to the scattering event is reduced.

The quantum-mechanical counterpart of this system, in general, shows ample oscillations in the conductance as a function of the external parameter γ, such as gate voltage or a magnetic field. This feature originates from a series of resonance overlaps.

Assume that for a stationary state vector of incident wave with channel n—say, $|\Psi_n^i\rangle$—a state vector of final wave—say, $|\Psi^f\rangle$—uniformly wanders the d-dimensional unit hypersphere S_0 in Hilbert space as a function of γ. This requirement is reasonable if the scattering process inside a cavity is intricate enough to receive a statistical nature into $|\Psi^f\rangle$. The statistical nature implys ergodicity on S_0, while the magnitude of d shows the complexity of the topological structure or richness of the information involved in the scattering process and is expected to be connected with the non-integrability of the system. $|\Psi^f\rangle$ is considered as an eigenvector of a given Hamilton operator H. Because each H eigenvector uniformly covers S_0 as H moves depending on γ, H can be described by an orthogonally invariant ensemble ($\beta = 1$) in the presence of time-reversal symmetry and a unitary invariant ensemble ($\beta = 2$) in the absence of such symmetry. Therefore, $|\Psi^f\rangle \in \mathbf{R}^d$ for $\beta = 1$ and $|\Psi^f\rangle \in \mathbf{C}^d$ for $\beta = 2$.

For simplicity, we consider the case in which N, the maximum number of transverse modes (channels) of propagating waves inside the leads, equals 1. Experimentally, this situation is more accessible when the Fermi energy of the electron is low enough, or the width of the leads is relatively small, i.e. the system is weakly open. Then the conductance is

$$G \propto T = |\langle\Psi_{m=1}^f|\Psi_{n=1}^i\rangle|^2 \,. \tag{1}$$

Here, the inner product is interpreted as a projection of the d-dimensional eigenvector $|\Psi_{m=1}^f\rangle$ to a fixed axis $|\Psi_{n=1}^i\rangle$. As employed in nuclear physics [13], a joint probability distribution for the d components of the eigenvectors reads

$$P_\beta(x_1, x_2, \ldots, x_d) = c_\beta \cdot \delta\left(\sum_{i=1}^{d} |x_i|^2 - 1\right) \tag{2}$$

where the normalization constants are $c_1 = \pi^{-d/2}\Gamma(d/2)$ and $c_2 = \pi^{-d}\Gamma(d)$. Simply integrating out all the components other than $x = \langle\Psi_{m=1}^f|\Psi_{n=1}^i\rangle$ yields the reduced densities

$$P(x) = \pi^{-1/2}\Gamma(d/2)\Gamma\left(\frac{d-1}{2}\right)^{-1}(1-x^2)^{(d-3)/2} \qquad (\beta = 1) \tag{3}$$

and

$$P(x) = \pi^{-1}(d-1)(1-|x|^2)^{d-2} \qquad (\beta = 2)\,. \tag{4}$$

After transformation of the variable x to $T = |x|^2$, we get analytical results for the distribution of the transmission coefficient for $N = 1$ as

$$P(T) = \frac{\Gamma\left(\frac{d}{2}\right)}{\sqrt{\pi}\,\Gamma\left(\frac{d-1}{2}\right)} T^{-\frac{1}{2}}(1-T)^{\frac{d-3}{2}} \qquad (\beta = 1) \tag{5}$$

and

$$P(T) = (d-1)(1-T)^{d-2} \qquad (\beta = 2)\,. \tag{6}$$

In the limit of $d \to \infty$, the distribution (5) goes to the so-called Porter–Thomas distribution, which is proposed to describe the probability distribution for the widths of resonances, in a nuclear reaction [14].

We should notice that for $N = 1$ the T distributions obtained by a random S-matrix theory for chaotic scattering [10] correspond exactly to equations (5) with $d = 3$ and (6) with $d = 2$ in the presence and the absence of time-reversal symmetry, respectively. However, our theory applies not only to those ideal cases but to a wide variety of open systems including regular cavities. The parameter d is considered to serve as an index of the degree of complexity accompanying fluctuations of the transmission coefficient and reflects the nature of the underlying classical dynamics. Here, d should no longer be an integer and can take any non-negative real number. For $N \geqslant 2$, it is not so easy to get a distribution function of T. The difficulty is that there could be a strong correlation between components $\{\langle \Psi_m^f | \Psi_n^i \rangle\}_{m,n=1,2,\ldots,N}$ and hence we cannot treat each of them equally. Here we do not go into details in this case.

From equations (5) and (6), we obtain the average and variance as

$$\left.\begin{aligned} \langle T \rangle &= \frac{1}{d} \\ \langle \delta T^2 \rangle &= \frac{2(d-1)}{d^2(d+2)} \end{aligned}\right\} \qquad (\beta = 1) \tag{7}$$

and

$$\left.\begin{aligned} \langle T \rangle &= \frac{1}{d} \\ \langle \delta T^2 \rangle &= \frac{d-1}{d^2(d+1)} \end{aligned}\right\} \qquad (\beta = 2)\,. \tag{8}$$

Now, we shall compare the theory with numerical results and determine the number d. We have computed the electric conductance for weakly open circle and stadium billiards (insets of figure 1) in the absence of a magnetic field. Geometries of the billiards are the same as those adopted in [5], but the width W of the leads is one half of those in [5]. We should note that the arrangement of the leads considerably reduces a contribution of direct transmission of electrons to the conductance. As γ, we choose $k_{\rm F}$, i.e. the Fermi wavenumber of electrons. The numerical distribution $P(T)$ was obtained by sampling 2512 values of $k_{\rm F}$ in $k_{\rm F} W/\pi \in [1, 2)$ for each billiard, and the interval $\delta k_{\rm F}$ of the nearest-neighbour values is much smaller than a width of each resonance.

Figure 1 shows that the theoretical predictions of $P(T)$ successfully describe the numerical results for both regular (circle) and chaotic (stadium) billiards. In each case the value d in (5) was extracted directly from the numerical data: $d = 2.5$ for the circle and $d = 4.0$ for the stadium. These values of d are not altered if we choose W twice for a width of the leads. This reveals a strong presumption that, though the explicit relation between d and the Liapunov exponent is not clear, d takes a larger value for scattering through a chaotic than a regular cavity; that is, d is specific to each system and displays a degree of non-integrability of the underlying classical dynamics. In the case of stadium billiards, d is greater than 3, which suggests that there exists a chaotic scattering process that has more complexity than is described by the circular ensemble of random S-matrix theory. In the case of circle billiards, the wavefunction pattern in the cavity shows radial chains of mountains (see the inset of figure 1(a)), corresponding to the asterisk classical orbits [5]. The adiabatic

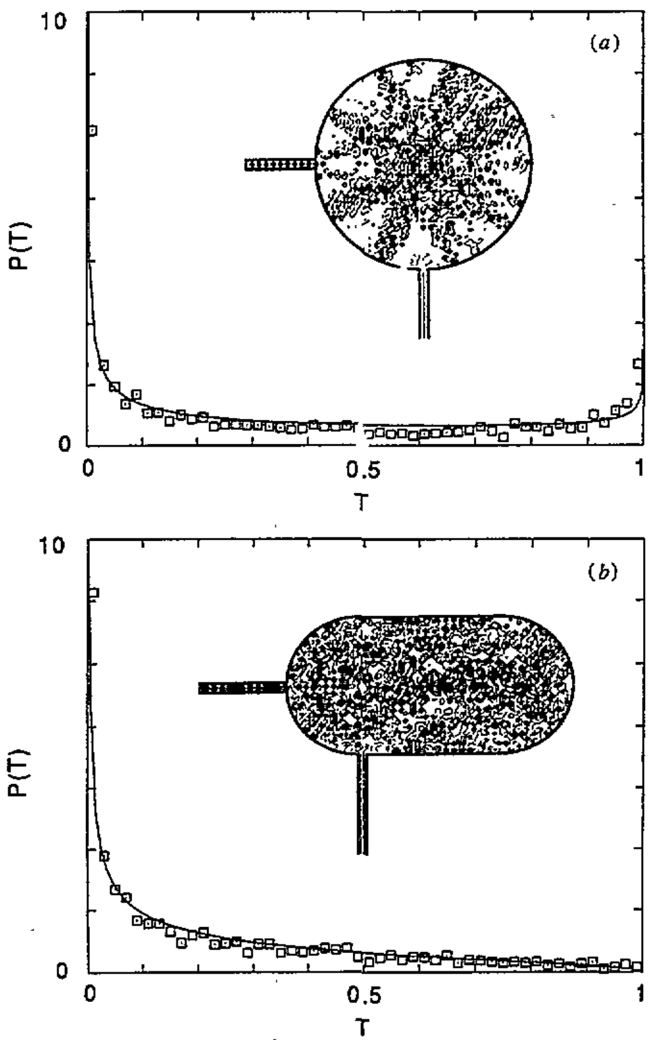

Figure 1. Distribution of the transmission coefficient for $N = 1$ in (*a*) circle and (*b*) stadium billiards with a pair of conducting leads whose orientation is the same as in [5]. The width W of the leads is $W/\sqrt{A} = 0.0468$, where A is the area of cavity region. The squares are the numerical results; the curves are the theoretical predictions with (*a*) $d = 2.5$ and (*b*) $d = 4.0$. The numerical results are obtained by shifting a value of k_F by $\delta k_F \cdot W/\pi = 0.000\,398$ within the full range of $N = 1$. Insets: the geometry of weakly open billiards and the typical density distribution of electrons with $k_F W/\pi = 1.591\,55$.

change of k_F causes a rotational motion of the pattern along the circular boundaries. This two-dimensional motion contributes the main part of the geometrical dimensions d obtained for the circle billiard. The remainder of d (~ 0.5) might be caused by the addition of diffraction around openings of the cavity, as was pointed out in another statistical argument of the conductance fluctuations [15]. This additive dimension is also included in d for the stadium billiard.

In the following we list the numerical results of the average and the variance of T, with the theoretical values in (7) (round brackets). For circle billiards, $\langle T \rangle \simeq 0.38$ (0.4) and $\langle \delta T^2 \rangle \simeq 0.13$ (0.11); for stadium billiards, $\langle T \rangle \simeq 0.23$ (0.25) and $\langle \delta T^2 \rangle \simeq 0.07$ (0.06). Here, we see good agreement between theory and the numerical results for both $\langle T \rangle$ and $\langle \delta T^2 \rangle$, which are smaller for the stadium than for the circle billiard. Especially, $\langle \delta T^2 \rangle$ for the stadium billiard is suppressed to almost a half of that for the circle billiard. The features of $\langle T \rangle$ and $\langle \delta T^2 \rangle$ are attributed, respectively, to weak localization and universal conductance fluctuations in chaotic scattering, as is commented in [5].

As an external parameter γ, we can choose a magnetic field, if it exists, as well as k_F. In that case, however, the range of parametric change should be small so as not to essentially alter the structure of the phase space.

In conclusion, we have shown the existence of geometrical dimensions characterizing scattering properties for a wide variety of mesoscopic conducting cavities. We have derived analytical expressions for the distribution of conductance in the lowest transmittable mode. The expressions include a parameter d which corresponds to the number of geometrical dimensions extractable from experimental data of conductance fluctuations. We propose that the magnitude of d is related to the non-integrability of a system and our numerical results for regular and chaotic open billiards exemplify this proposition.

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